Introduction to Cognitive Science for Mathematical Scientists

Cognitive Science 050.313/613

Hodgkin-Huxley Theory: The Action Potential

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Hodgkin-Huxley Equation: Membrane current, J_m

$$\frac{a}{2R_i}\frac{\partial^2 V}{\partial x^2} = J_m = C_m\frac{\partial V}{\partial t} + \sum_{i=\mathrm{K,Na,L}} g_i(V,t)[V-E_i]$$

Solve numerically for V

Excellent empirical fit to Action Potential in squid giant axon

feeds

feeds

Cable Equation: $\partial V/\partial x$

$$\frac{a}{2R_i}\frac{\partial^2 V}{\partial x^2} = J_m$$

Parallel conduction model: $\partial V/\partial t$

$$J_m = C_m\frac{\partial V}{\partial t} + \sum_{i=\mathrm{K,Na,L}} g_i(V,t)[V-E_i]$$

Basic Circuit Equations: Ohm's Law, etc. ①

$I = gV$; $I = C\,\partial V/\partial t$; $g_{\text{parallel}} = g_1 + g_2 + g_3$

entails

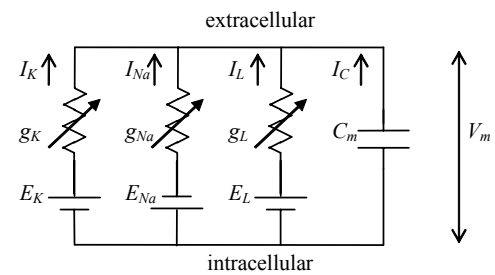
feeds

feeds

Ohm's Law: Axial current I_i

$$-\frac{\partial V}{\partial x}\Delta x = -\Delta V_m = I_i R_i^{\Delta x}$$
$$= I_i(R_i\Delta x/A) = J_i R_i \Delta x$$

Equivalent circuit



H.W. 9-12.2

Kirchoff's Law: Axial + Membrane Currents

$$0 = \frac{\partial I_i}{\partial x}\Delta x + I_m = \frac{\partial J_i}{\partial x}A\Delta x + J_m 2\pi a\Delta x$$

H.W. 9-12.3

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Hodgkin-Huxley Gate Model: Conductances g_i

$$g_{\mathrm{K}}(V,t) = \bar{g}_{\mathrm{K}} n(V,t)^4$$
$$g_{\mathrm{Na}}(V,t) = \bar{g}_{\mathrm{Na}} m(V,t)^3 h(V,t)$$

K gates: 4 particles n activated by depolarization

Na gates: 3 particles m activated by depolarization

1 particle h de-activated by depolarization

Nernst Equation: Equilibrium Potentials E_i

$$E_i = \frac{kT}{ze}\ln\left(\frac{[X_i]_{\text{out}}}{[X_i]_{\text{in}}}\right)$$

Ex. 2.2:20

H.W. 9-12.1

entails

feeds

First-order gating particle kinetics: State dynamics $n(t)$

$$\text{gate open state: } n(t) \underset{\alpha_n(V)}{\overset{\beta_n(V)}{\rightleftharpoons}} [1-n(t)]\text{: gate closed state}$$

$$\frac{dn(V,t)}{dt} = \alpha_n(V)[1-n(V,t)] - \beta_n(V)n(V,t)$$

sim. for m, h

Nernst-Planck Equation: Passive J_m

$$J = -\left(\mu z[X]\frac{\partial V}{\partial x} + \frac{\mu kT}{e}\frac{\partial [X]}{\partial x}\right)$$

feeds

feeds

Drift flux $\mathbf{J}_{\text{drift}}$

$\mathbf{J}_{\text{drift}} = -\mu\, z\,[Y^z]\,\nabla V$

Rate parameters

$\alpha_n^0, \beta_n^0, \gamma ze/kT$

fit empirically from solutions for clamped V; g measured as I/V (Ohm's Law obeyed)

feeds

Energy Barrier Model: $\alpha(V)$, $\beta(V)$

$$\beta_n(V) = \beta_n^0 e^{-(1-\gamma)zeV/kT}$$
$$\alpha_n(V) = \alpha_n^0 e^{\gamma zeV/kT}$$

Fick's Law; Einstein: Diffusion flux $\mathbf{J}_{\text{diff}}$

$\mathbf{J}_{\text{diff}} = -(kT/e)\,\mu\,\nabla[Y^z]$